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ATTITUDE DETERMINATION OF A CUBE SATELLITE USING SUN SENSORS

Bachelor's thesis

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KUUBIK-SATELLIIDI ASENDI MÄÄRAMINE PÄIKESESENSORITEGA

Bakalaureusetöö

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Tallinn 2017

Author's declaration of originality

I hereby certify that I am the sole author of this thesis. All the used materials, references to the literature and the work of others have been referred to. This thesis has not been presented for examination anywhere else.

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Abstract

The aim of this bachelor's thesis is to determine satellite's attitude in orbit using sun sensors.

Satellite will be launched into space to take pictures of the Earth. To be able to do it, we must know, what is the determination of the satellite. Using sun sensors is one possibility to find the determination.

The thesis provides an overview of the sun sensors on the satellite, different coordinate systems and reference frames. In the third chapter, position of the Sun and vector are calculated. In the forth and last part, rotating a vector using quaternions is implemented. The results are achieved by using Matlab.

The thesis is in English and contains 39 pages of text, 5 chapters, 15 figures, 3 tables

Annotatsioon

Kuubik-satelliidi asendi määramine päikesesensoritega

Käesoleva lõputöö eesmärgiks on määrata satelliidi asend orbiidil, kasutades päikesesensoreid. Selleks arvutatakse päikesevektor satelliidi suhtes, mis igal ajahetkel (0.1s) annab teada, kus asub päike satelliidi kolme telje suhtes. Päikesevektori arvutamiseks antakse ette satelliidi koordinaadid orbiidil ning kuupäev ja kellaaeg. Tulemusena saadakse päikesevektor.

Satelliidil asuvad päikesesensorid, mis mõõdavad samuti päikese asendit sensorite suhtes.

Seejärel saab võrrelda arvutatud ja mõõdetud päikesevektoreid omavahel, mille tulemusel saame teada, kus asub Maa satelliidi suhtes. Satelliidi eesmärk orbiidil on teha Maast pilte, mistõttu peame teadma, millal on satelliit sellises asendis, et sateliidi ühel tahul asuv kaamera on suunaga Maa poole.

Tulemuse täpsuse eesmärgiks on 1°.

Töö sisaldab teoreetilist ülevaadet sensoritest satelliidil ning täpsemalt kirjeldatakse päikesesensoreid. Samuti antakse ülevaade tähtsamatest koordinaat- ja taustsüsteemidest, mida on vaja satelliidi asendi määramiseks. Praktiline osa sisaldab päikese asendi arvutamist, mille käigus leitakse päikese asukoht kindlal ajal kindla koha suhtes. Seejärel saab leida päikesevektori, mida saab kasutada töö viimases praktilises osas. Viimane osa sisaldab vektorite pööramist kvaternioonide abil, mille tulemusena saab teada, kuidas tuleb satelliiti pöörata, et kaamera oleks suunatud Maa poole.

Päikese asendi ning teiste arvutuste jaoks on kasutatud Matlab programmi.

Lõputöö on kirjutatud inglise keeles ning sisaldab teksti 39 leheküljel, 5 peatükki, 15 joonist, 3 tabelit.

List of Abbreviations and Terms

ADCS	Attitude Determination and Control System
Altitude	Latitude in horizon coordinate system
AU	Astronomical Unit, around $1.496*10^{11}$ metres
Azimuth	Longitude in horizon coordinate system
BC	Before Christ
Cross product	Mathematical operation on two vectors in three-dimensional space. Perpendicular to both vectors
Declination	Latitude in equatorial coordinate system
Earth Albedo	The fraction of solar energy reflected from the Earth back into space
ECEF	Earth Centered Earth Fixed frame
ECI	Earth Centered Intertial frame
Ellipse	Oval in shape, of which a circle is a special case
Equation of Center	The angular difference between true and mean anomaly
ESTCUBE-2	Estonian satellite project
Gimbal lock	In three-dimensional system, where two axes align
Hour angle	The difference between the local sidereal time and the RA
Julian Date	The number of days elapsed since January 1 4713 BC.
MATLAB	Mathematical computing software for engineers and scientists
Mean anomaly	The true anomaly, if the Earth moved along a circular orbit
NESW	North-East-South-West
Perihelion	The point in the orbit of a planet nearest to the Sun
RA	Right ascension, longitude in equatorial coordinate system
Right-hand system	Set of three axes, labelled so that rotates from the positive x-axis towards the positive y-axis towards the positive z-axis
Sidereal time	Time measured with respect to the motion of the stars
Solar transit	The moment at which a celestial body crosses the observer's meridian
True anomaly	The angle between the Earth and the perihelion of the orbit of the Earth
TTÜ	Tallinn University of Technology
Vernal equinox	The moment at which the Sun crosses the celestial equator (RA=0°)

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1 Introduction

Nanosatellites gain more popularity with time around the world.When 10 years ago only 9 satellites were launched into space, then in 2016 the number of satellites launched into space increased to 86 [1]. And in the first 4 months of this year, the number of satellites launched to space has exceeded 2016’s total and it’s plan to launch around 500 more satellites this year [1].

According to the website [2], in the fall semester of 2014, TTÜ Innovation and Business Centre Mektory started with a Satellite Programme directed to students and professors in collaboration with engineering and space technology industries from Estonia and other countries. The mission of the programme is to provide high-quality workforce in high-tech companies in Estonia and other countries. The goals of the programme are to send reliable cube-satellite into space, that is possible to operate in earth station. Also to give students the opportunity to practice themselves in that field. Building and testing the satellite takes place in 2016-2017 and launching the satellite most probably in 2018.

The cube satellite will be built according to CubeSat design. CubeSat is a 10x10x10 cm cube shaped satellite having a mass with 1.33 kg [3].

The purpose of the satellite in space is to take pictures of Earth while moving in orbit. To be able to take pictures, satellite’s camera has to be pointed to Earth.

As Estonia is turning 100 in 2018, in collaboration with ESTCUBE-2, an Estonian song chosen by the people will be saved to the satellite’s memory and along with the satellite, will be launched into space [4].

1.1 Progress in this work

The purpose of this work is to determine satellite’s attitude using sun sensors. First, we have to calculate the sun vector with respect to the satellite’s position. Then it is possible to compare calculated vector and vector from sun sensors and after rotating calculated vector as needed, it is possible to find what is the attitude of the satellite. The program’s inputs are timestamp and satellite’s coordinates and the output is a quaternion representing the rotation.

The work is divided into three parts. First part is an overview of sun sensors and their working principles. In the second chapter, sun vector is calculated and in the third and final part, the position of the Earth as seen from the satellite is calculated.

2 Attitude determination and Control System

To get a satellite's position in orbit with its rotational speed and attitude, attitude determination and control system (ADCS) [5] is used. It means that satellite is equipped with sun sensors, magnetometers, and gyroscopes. We get an information from these sensors and can make adjustments to get desired attitude and rotational speed.

2.1 Sensors

Magnetometer sensors [5] measure the intensity of magnetic field in respect to the three axes of the satellite on this basis. The vector gives us an information about the direction of the magnetic field in relation to the satellite. Comparing the calculated vector and the onboard Earth magnetic field model, it is possible to calculate necessary currents to direct the satellite.

Gyroscopes [5] are important to the satellite to determine the satellite's rotational speed in relation to all three axes.

2.1.1 Overview of sun sensors

This work concentrates more on sun sensors, how to determine satellite's attitude using sun sensors.

Sun sensors determine the Sun's position in relation to the satellite. We need to find the Sun vector in all three axes. The satellite has 6 sun sensors, which means one sensor in every side (Figure 1). There are three types of sun sensors, which are being introduced – analog sun sensor, digital sun sensor and sun panels. Every sensor gives us the intensity of the light signal. Depending to the angle of the sun ray falling to the sensors, it is possible to calculate how much power the sun panels on a satellite can produce. There can be a problem when the sensors receive light from the Earth. For that problem, Earth Albedo model is used, which will be discussed in the following chapter.

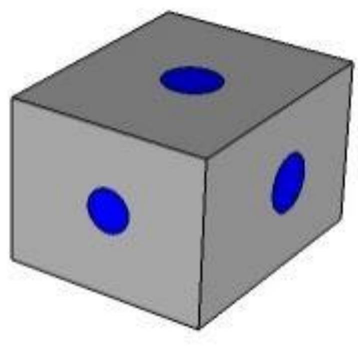

Figure 1. Position of the sun sensors **[6]**.

2.1.2 Analog sun sensor

An analog sun sensor measures the sun's intensity, but not the direction of the Sun. The measure is calculated by the energy flux that goes through the surface area of the photocell (Figure 2). The angle in analog sun sensor can be calculated from the current:

$$I_e = I_{max}\cos\theta$$

where I_{max} is the maximum current generate in photocell, I_e is the current from the Sun[7].

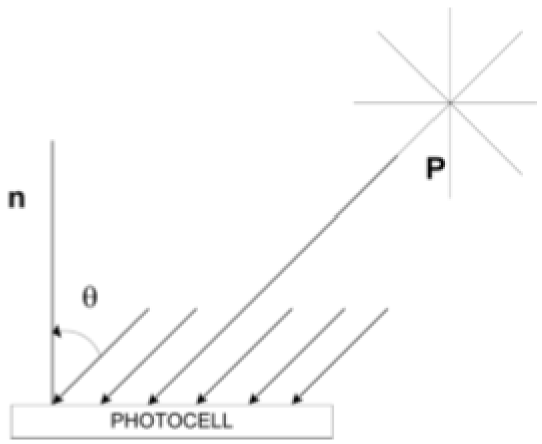

Figure 2. Analog sun sensor**[7]**.

2.1.3 Digital sun sensor

A digital sun sensor measures in addition to the Sun's intensity, the direction of the Sun. The digital sun sensor is built of a pattern of photocells, where the incoming sunlight shows from which direction the rays are from (Figure 3) [7].

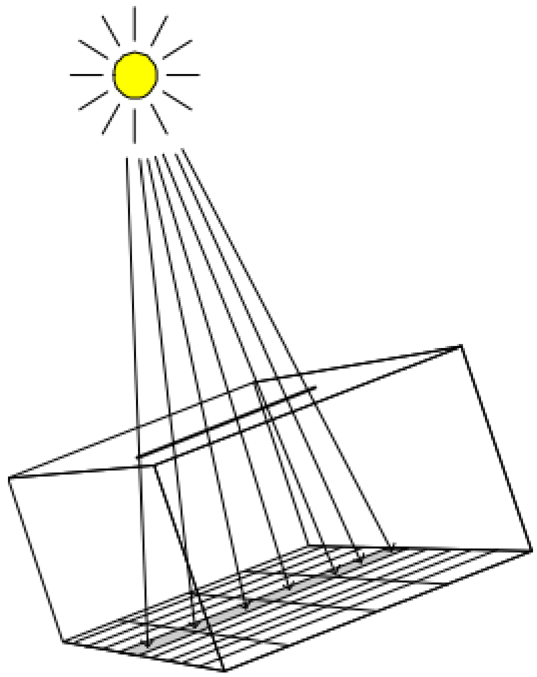

Figure 3. Digital sun sensor [7].

2.1.4 Sun panels

Sun panels on the satellite only produce power from the Sun. It is not possible to get the Sun's angle nor direction.

2.2 Earth Albedo model

While satellite orbits around the Earth, the sun panels on the satellite might not only get light from the Sun, but also from the Earth. That is called Earth Albedo error. Depending on the position of the satellite with respect to the Earth, satellite might get more or less light reflected from the Earth. For example, sandy and polar areas reflect much more sunlight than oceans and forests [8]. This work doesn't include Earth Albedo model, but should be implemented in future work.

3 Calculating the sun vector

3.1 Coordinate systems

To find a position to any astronomical object, there must be a coordinate system. There are many different coordinate systems that differ by the plane of the system. Coordinates have two numbers, first of them refers to „how far round“ and the second coordinate „how far up“. In the next section, we are going to give a short introduction to the three satellite coordinate systems that will be used in calculating the attitude of the satellite.

3.1.1 Horizon coordinates

The position of the celestial object is relative to the observer’s horizon [9] . The observer’s horizon is the circle NESW (North-East-South-West) as seen from Figure 4. Azimuth and altitude are in use in horizon coordinate system. Azimuth means „how far round“ from 0° to 360° and altitude „how far up“ from 0° to 90° and they are both measured in degrees. This system isn’t good for positioning stars because the system moves as the Earth rotates and it is hard to fix the star’s position [10]. But finding the Sun’s position, this system is suitable, because the coordinates are dependent on the observer’s longitude and latitude – that’s exactly what we want.

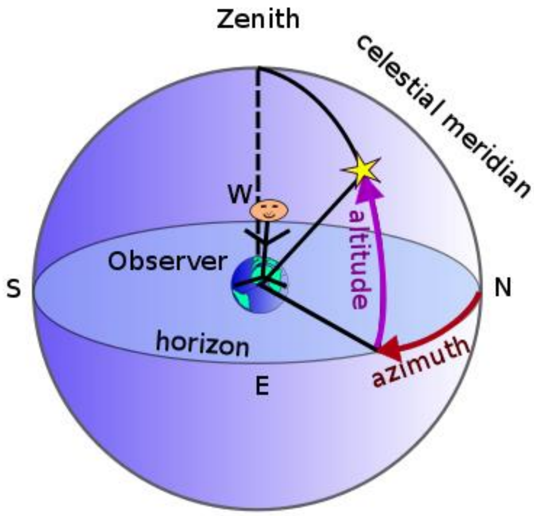

Figure 4. Horizon coordinate system [9].

3.1.2 Equatorial coordinates

In this coordinate system, the coordinates refer to the plane of the celestial equator (Figure 4) [11]. The coordinates in the equatorial coordinate system are called right ascension, α, and declination, δ. These coordinates are ideal to describe the position of the stars and „fixed“ heavenly bodies because declination and right ascension are constants [11]. Although, for the Sun, Moon, and planets they change with time. Right ascension is measured in hours, minutes and seconds and declination in degrees, where positive degree signifies to the north of the equator and negative to the south of the equator. The full circle is made in 24 hours of sidereal time. Right ascension is also related to hour angle, H, which is as right ascension the coordinate of „how far round“. The angle indicates, how far the star has travelled from the south meridian point along the equator [14]. As measured right ascension, hour angle is also measured in hours, minutes and seconds. They both can also be converted into degrees between 0°-360° by multiplying by 15. 1h equals to 15°. This system gives more precise results compared to the horizontal coordinate system, because the full circle is one year, where in the horizontal coordinate system, it is one day and therefore the results don't change much in time.

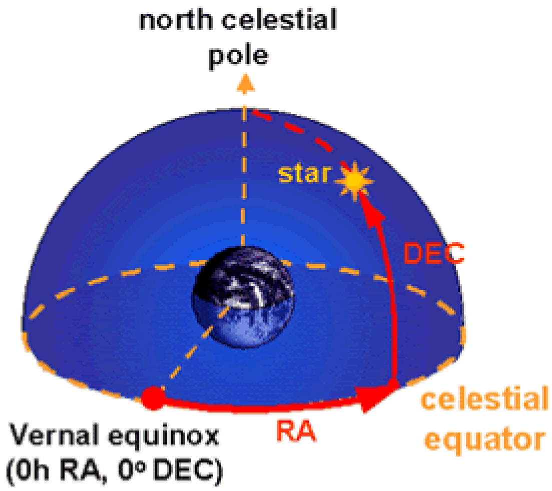

Figure 5. Equatorial coordinate system [11].

3.1.3 Ecliptic coordinates

The plane in ecliptic coordinate system is Earth's orbit around the Sun, also called ecliptic [11]. When calculating the position of the object in the Solar System, the best coordinate system is ecliptic. As with equatorial coordinate system, ecliptic also uses vernal equinox as its reference direction, where around on March 21 both ecliptic longitude (right ascension in equatorial) and ecliptic latitude (declination in equatorial) are 0°. The circle is made in 1 year, as it is in the equatorial coordinate system.

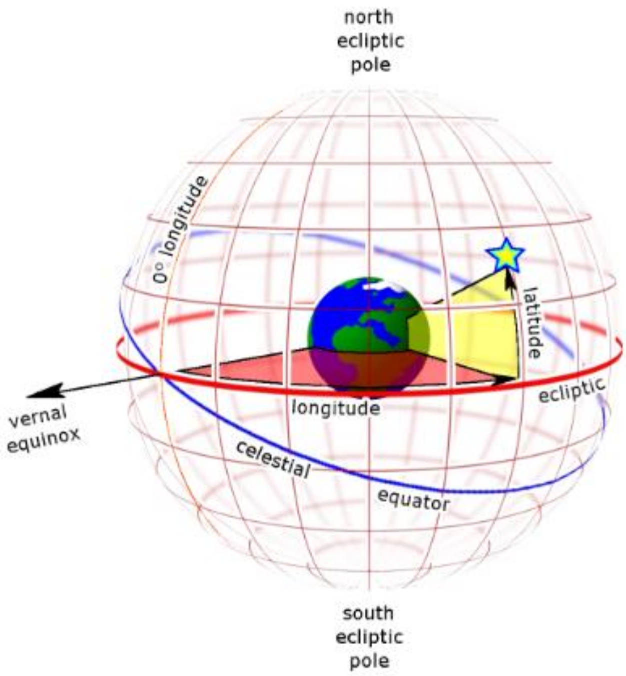

Figure 6. Ecliptic coordinate system [12].

3.2 Reference frames

The satellite must be described by reference frame to be able to orient the satellite. Reference frame is a three-dimensional coordinate system, which gives an information about the position of specific object in the chosen coordinate system. There are several reference frames, some of them are described in the following sections.

3.2.1 Earth Centered Inertial (ECI) frame

The origin of the frame is at the center of earth, but it doesn't rotate with the Earth [13]. As seen from the Figure 7, the coordinates are following:

- x-axis points to the vernal equinox direction
- y-axis completes the right-hand system
- z-axis points upwards to the north pole

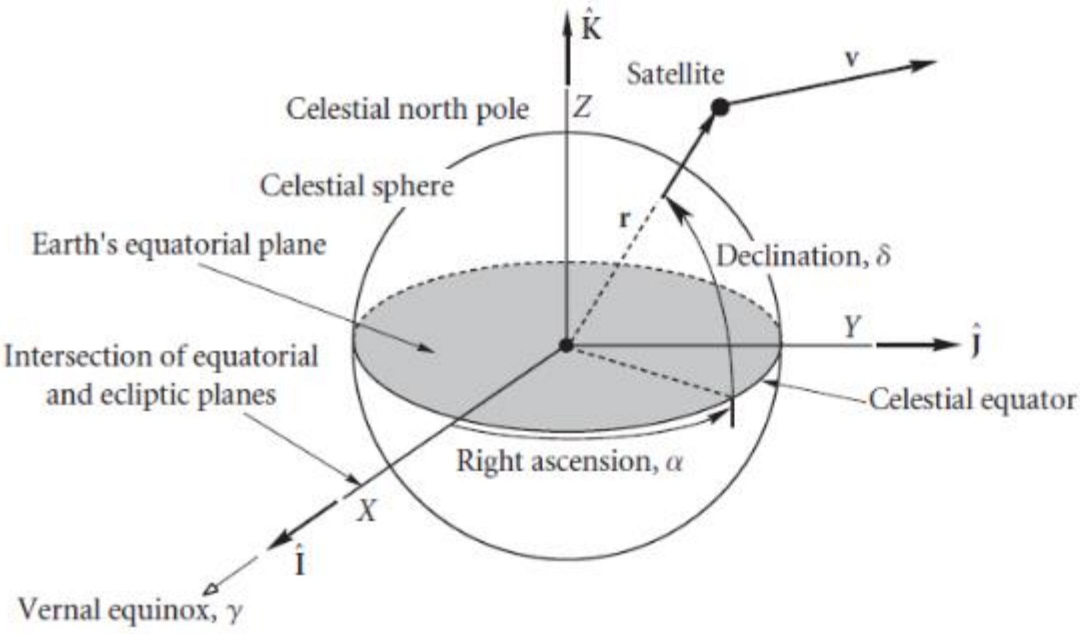

Figure 7. ECI frame [13].

3.2.2 Earth Centered Earth Fixed (ECEF) frame

It is similar to ECI frame. The only difference is that this is Earth Fixed – means that the origin of the frame rotates along the Earth [13]. As seen from Figure 8, the coordinates are following:

- x-axis points from the center of the Earth to the intersection of the meridian and equator
- y-axis completes the right-hand system
- z-axis points upwards to the north pole

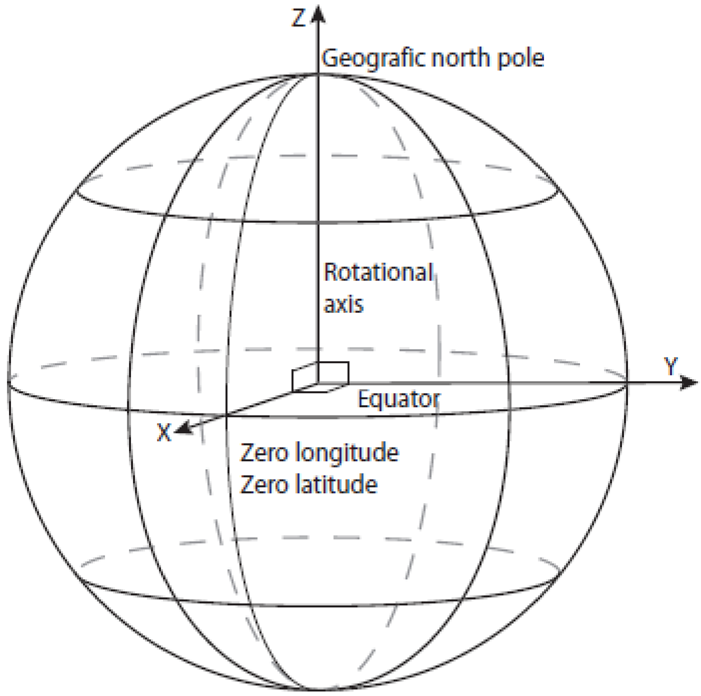

Figure 8. ECEF frame [13].

3.2.3 Orbit Reference frame

The origin of the orbit reference frame is the center of the satellite's mass [13]. As seen from Figure 9, the coordinates are following:

- x_R-axis points to the direction of satellite's motion
- y_R-axis completes the right-hand system
- z_R-axis points to the center of the Earth

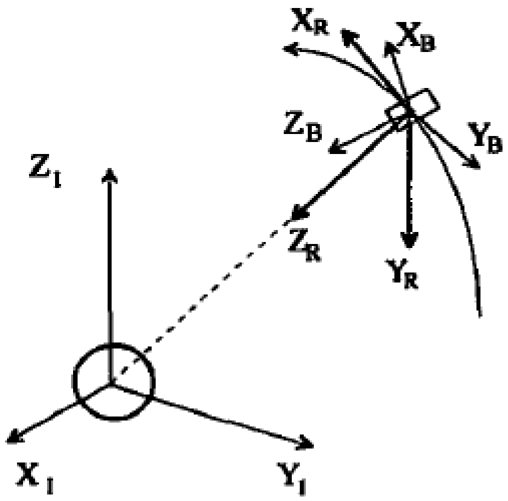

Figure 9. Orbit frame [13].

3.2.4 Satellite Body frame

The origin of the satellite's body frame is the center of the satellite's mass [13]. This frame is used to describe the satellite's attitude. As seen from Figure 10 the coordinates are following:

- x-axis points from the back to the front
- y-axis completes the right-hand system
- z-axis points from the top to the bottom

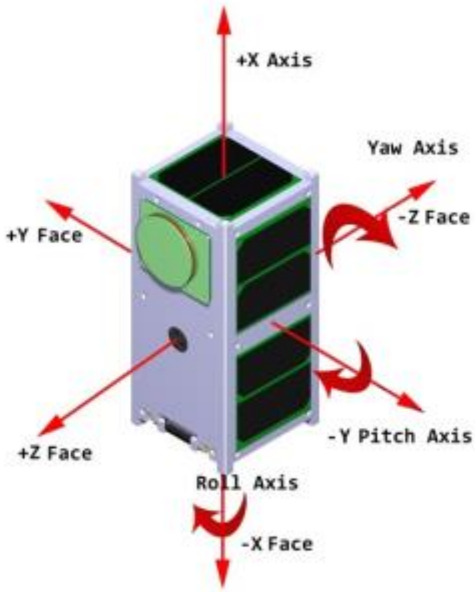

Figure 10. Body frame [13].

3.3 Calculating the position of the Sun

Our inputs are date and time, for example, 11/05/2017 18:00:00 and longitude and latitude, where we would like to know about the Sun's position. In the following example, our longitude is lon=59.395884 and latitude lat=24.671431 which are the coordinates of the Tallinn University of Technology. In the following example, Astronomy Answers: Position of the Sun [14] is the main reference for finding the position of the Sun.

As we know, Earth orbits around the Sun, but to find the Sun's position, it is easier to think that the Sun moves around the Earth.

In astronomy, the date has been converted into Julian Date. The purpose of Julian Calendar is to make easy calculations between dates in different calendars [15]. Julian Day started on January 1 4713 BC at noon and will end January 22 3268 [10]. The formula for calculating a Julian Date at a specific date and time will be

year = 2017

month = 05

day = 11

hour = 18

minute = 00

second = 00

a = $\frac{14-month}{12}$

y = year + 4800 – a

m = month + 12*a – 3

$$J = day + \frac{153*m+2}{5} + 365*y + \frac{y}{4} - \frac{y}{100} + \frac{y}{400} - 32045$$

Rounding is down in calculating Julian Date

To want to add a time then

$$JD = J + \frac{hour - 12}{24} + \frac{minute}{1440} + \frac{second}{86400}$$

For our date and time, the Julian Day will be

a = 0

y = 6817

m = 2

J = 2457886

JD = 2357886.25

If the orbit were a perfect circle, then the Sun would move constantly around the Earth and it would be much easier to calculate the Sun's position. But we are going to imagine that it does move around the Earth in a circle rather than along the ellipse. The angle relative to perihelion (closest point to Earth) is called mean anomaly, M, and can be found

$$M = M_0 + M_1*(JD-J2000)$$

As the mean anomaly is an angle, we have to take modulo of 360°

M_0 and M_1 are constants and for the Earth, they can be found from Table 1

J_{2000} is an epoch that is a moment in time used as a reference point

J_{2000} = 2451545

M = 126.48°

We actually need to find the true anomaly, v, which is the motion of the Sun in an ellipse. The difference between true anomaly and mean anomaly is called the Equation of Center, C, and for the Earth it can be calculated

$$C \approx C_1*\sin(M) + C_2*\sin(2*M) + C_3*\sin(3*M)$$

where C_1, C_2 and C_3 are constants and for the Earth they can be found from Table 1

$C \approx 1.5208°$

As we can see, the difference isn't much between a circular and an elliptic orbit.

Now we can calculate the true anomaly

$$v = C + M$$

$v = 128°$

We are now able to calculate planet's ecliptical longitude λ seen from the sun. But first, as for finding a true anomaly, we have to find a longitude L for the circular orbit and it can be found

$$L = M + \Pi$$

Π is an ecliptic longitude of the planet and for the Earth, it can be found from Table 2.

$L = 230.4°$

And now we can find a longitude for the elliptic orbit

$$\lambda = L + C$$

$\lambda = 231.9°$

Whether we're looking Sun from Earth or Earth from the Sun, the difference is 180° which means that we have to add 180° to ecliptic longitude to get the Sun's longitude and it has to be between 0 and 360°.

$$\lambda_{sun} = \lambda + 180° \% 360$$

$\lambda_{sun} = 50.939°$

We now go to equatorial coordinate system to get right ascension α and declination δ.

$$\alpha = \lambda_{sun} + A2*\sin(2*\lambda_{sun}) + A4*\sin(4*\lambda_{sun}) + A_6*\sin(6*\lambda_{sun})$$

$$\delta = D_1*\sin(\lambda_{sun}) + D_{3*}\sin^3(\lambda_{sun}) + D5*\sin^5(\lambda_{sun})$$

A_1, A_3, A_5, D_1, D_3 and D_5 are constants and for the Earth they can be found from Table 2

$\alpha = 49.487$

$\delta = 18.243$

Right ascension is usually converted into hours, minutes and seconds so we have to divide it by 15 and get

α = 3h 17min 57s

Next, we have to find, is sidereal time. Sidereal time is a time-keeping system to locate celestial objects. Sidereal time is a little faster than solar time. 24h sidereal time is equal to 23h 56min 4s of solar time. Formula for calculating sidereal time is

$$\Phi = (\Phi_0 + \Phi_1*(JD-J_{2000}) - lon)$$

where Φ_0 and Φ_1 are constants and for the Earth can be found from Table 2

As for previous calculations, we have to get modular of 360°

$\Phi = 115.56°$

Hour angle, H, is the difference between sidereal time and right ascension.

$$H = \Phi - \alpha$$

Hour angle is also expressed in hours, minutes and second as it was for right ascension

H = 4h 24min 18s

We have now found everything to find the Sun’s ecliptic coordinates altitude, alt, and azimuth, az. The formulas are based on The formula for calculating altitude and azimuth are following

$$alt = \arcsin(\sin(\delta)*\sin(lat)+\cos(\delta)*\cos(lat)*\cos(H))$$

$$az = \arccos\frac{\sin(\delta)-\sin(lat)*\sin(alt)}{\cos(lat)*\cos(alt)}$$

To get the correct angle of the azimuth, then we have to find a solar transit.Right now, when the azimuth angle reaches to 180°, to the meridian, then the angle starts descending. So we have to find, when is the solar transit to keep the angle rising.

The formula for solar transit is

$$J_{transit} = J_x + J_1*\sin(M) + J_2*\sin(2*\lambda_{sun})$$

where J_1 and J_2 are constants and for the Earth can be found from Table 1, M and λ_{sun} can be found from previous calculations. J_x is an estimated solar transit and can be calculated

$$J_x = JD + J_3*(n-nx)$$

J_3 is a constant and for the Earth can be found from Table 1. JD can be found from previous calculations. nx can be calculated

$$nx = \frac{JD-J_{2000}-J_0}{J_3} - \frac{lon}{360}$$

J_0 is a constant and can for the Earth be found from Table 1. To get n, nx has to be rounded to the nearest number. Now we can calculate solar transit.

$J_{transit}$ = 2357886.06703

In May 11 2017 18:00:00 the altitude and azimuth of the Sun are

alt = 27.44°

az = 258.78°

	M0	M1	C1	C2	C3	Π	J0	J1	J2	J3
Earth	357.5291	0.98560028	1.9148	0.0200	0.0003	102.9373	0.0009	0.0053	-0.0068	1.00

Table **1.** Constants.

	A2	A4	A6	D1	D3	D5	Π	θ0	θ1
Earth	-2.4657	0.0529	-0.0014	22.7908	0.5991	0.0492	102.9373	280.1470	360.9856235

Table 2. Constants.

Knowing the Sun's altitude and azimuth, it is now possible to find the Sun vector.

3.4 Sun vector

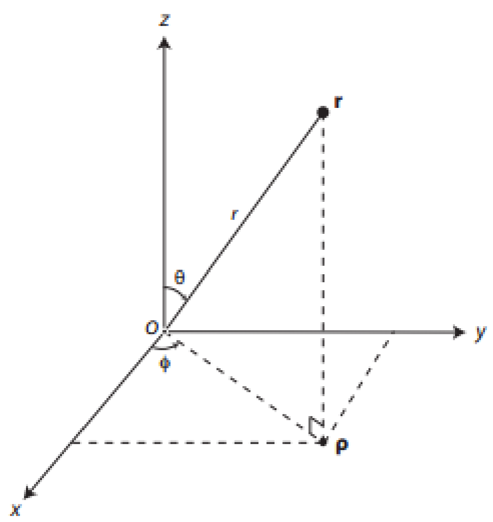

Figure 11. Vector in coordinate system [16].

r is desired vector with a distance, r, between Earth and the Sun. φ is the azimuth of the Sun and θ polar angle. As the distance between Earth and the Sun is close to 1 (r = 1.01AU), it's easier to leave r out. And for three-dimensional space, we get the sun vector [16]

$$r = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}$$

For our date and time used in finding the Sun's position, the Sun vector would be

$$r=\begin{pmatrix} -0.017482 \\ -0.86934 \\ 0.46226 \end{pmatrix}$$

3.5 Comparison

The comparisons are made for right ascension, declination, azimuth and altitude. Matlab result has been compared with different websites, which were:

Solar – Solar System Calculator [17]. It was able to get information about all of the compared angles.

SkyLive – The Sky Live [18]. It was able to get information about right ascension, declination, and altitude.

Wolfram – Wolfram Alpha [19]. It was able to get information about all of the compared angles. To find an information, there must be written: „sun position in [location] [time]"

SunCalc – Sun Calc [20]. It was able to get information about altitude and azimuth.

Spa – Spa [21]. It was able to get information about azimuth

The measurements are made on May 07 2017 from 15.45 to 16.15 with intervals of 15 minutes. As seen from the following figures, the results change constantly in time, so the result is taken at 15.45 in the following comparisons.

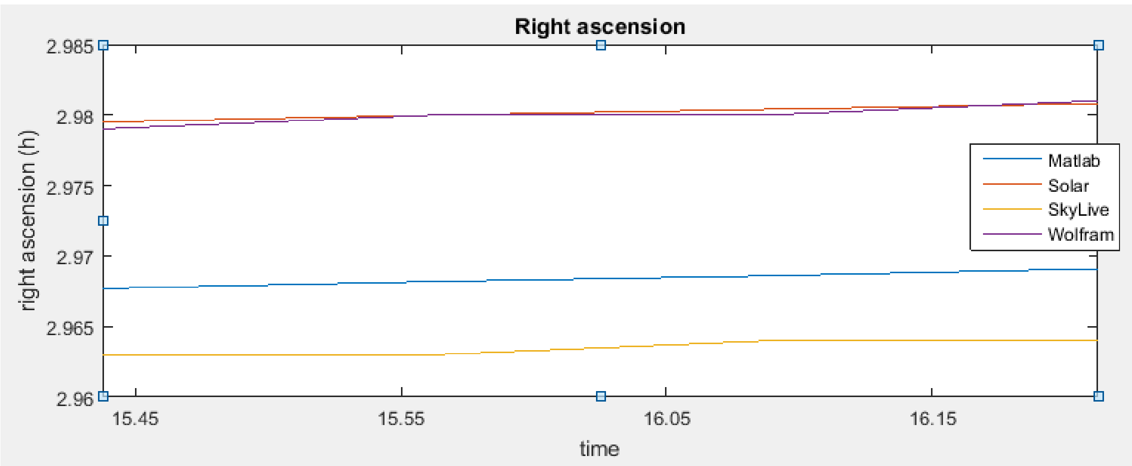

Figure 12. RA comparison.

As seen from Figure 12, the result of Matlab is about 2.968h. In comparison with other sources, the difference isn't much. Solar and Wolfram results are little less than 2.98. The difference with Matlab is 0.012h (0.18°). SkyLive result is less than Matlab's – 2.963 and the difference with Matlab is only 0.005h.

The difference between Matlab and other sources are very small. We can say that right ascension calculation is suitable and we can use it in other calculations.

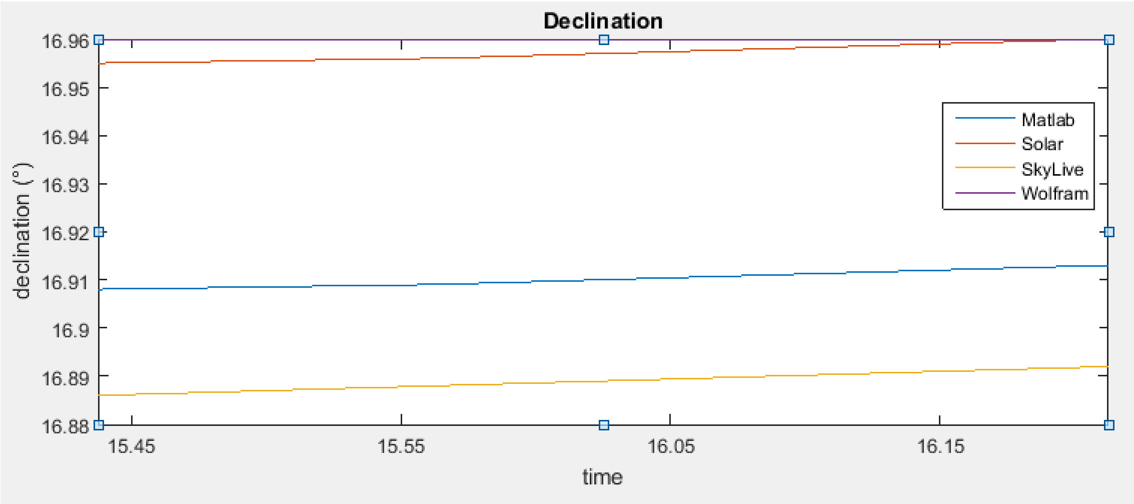

Figure 13. Declination comparison.

As seen from Figure 13, the Matlab result is about 16.91°. SkyLive gives about 16.882°, Solar 16.958° and Wolfram 16.96°. As seen from the Figure, the biggest difference comes to Wolfram – 0.05°.

The difference between Matlab and other sources is very small. We can say that declination is suitable.

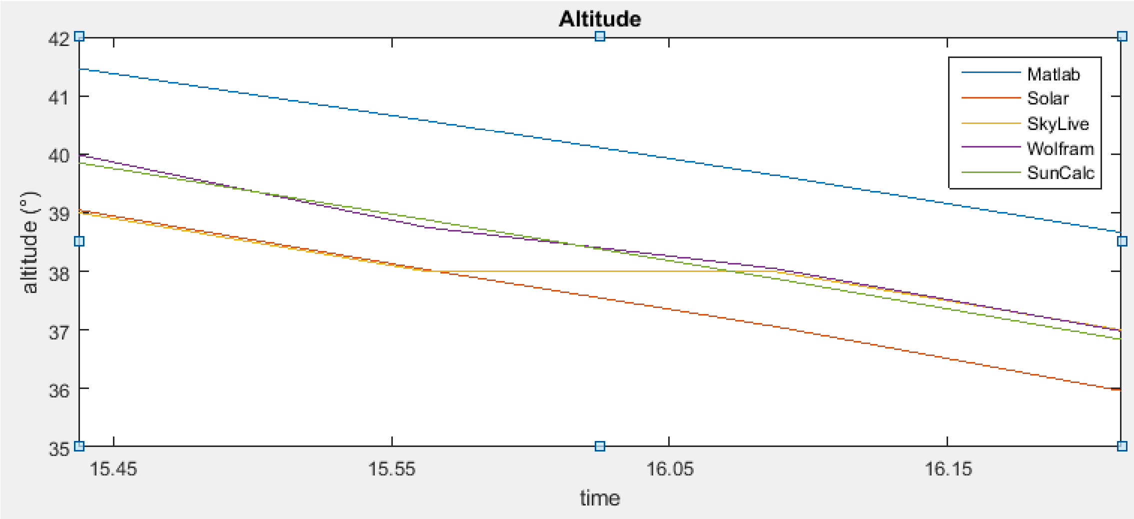

Figure 14. Altitude comparison.

As seen from Figure 14, Matlab gives a result of the biggest angle compared to other sources. It gives little less than 42°, around 41.6°. Wolfram and SunCalc have similar results - 40°. Solar and SkyLive give around 39°.

The difference between Matlab and other sources are about 2-3°. As the altitude changes quickly in one full day (in chapter 3.1.2), then this difference isn't much and Matlab result is suitable.

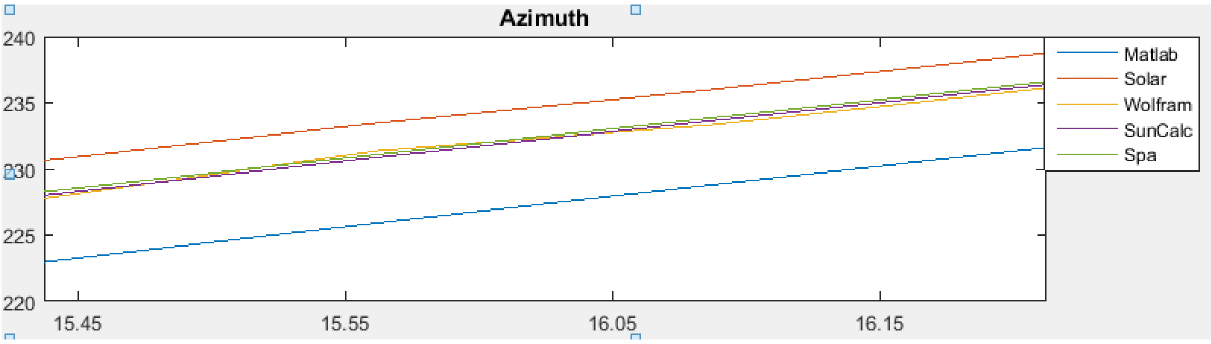

Figure 15. Azimuth comparison.

As seen from Figure 15, Matlab gives the smallest azimuthal angle – around 223°. Wolfram, SunCalc, and Spa all give almost the same result – around 228°. Solar gives around 231°.

The difference between Matlab and other sources is 5-8°. But as the azimuth changes quickly in one day, then the difference isn't much and the Matlab result is suitable.

The Matlab results with other sources are almost the same and the difference isn’t much. The difference in right ascension is 0.2%, in declination 0.15%, in altitude 5% and in azimuth 2.5%.

4 Attitude determination

Calculated from the previous chapter, we know what is the position of the Sun in relation to the center of the satellite. In addition, we get information about satellite's three axes from the sun sensors as introduced in chapter 2.2. The final step is to find out where is Earth with respect to the satellite. For finding the vector, we have to compare two given vectors and using quaternions, can calculate the third vector which represents the position of the satellite in relation to Earth.

4.1 Quaternions

Quaternions are used to rotate satellite's attitude and to convert vector from one reference frame to another, for example from body frame to orbit frame [22].

Quaternion is the best orientation formation to determine satellite's attitude, because there might be situations, where the sunlight doesn't fall to the satellite appointed and there might be gimbal lock. Gimbal lock happens, when two axes in three-dimensional system coincide [23]. Gimbal locks appear in other orientation formation, for example in Euler angles, but not in quaternion.

Quaternions in this work are used to rotate one vector to desired position using the other vector.

One vector in quaternion computation is a vector measured from sun sensors. But we don't have real measurements, so in the following example, two vectors has been taken just to find out if the computations in finding quaternion are acceptable and can be used in real system.

In this example, the two vectors are:

$$p1 = \begin{pmatrix} -0.17482 \\ -0.86934 \\ 0.46226 \end{pmatrix}$$

$$p2 = \begin{pmatrix} -0.86934 \\ -0.17482 \\ 0.46226 \end{pmatrix}$$

p1 is a sun vector calculated 11/05/2017 18:00.

Next, we have to find an angle between these vectors. It can be found:

$$\alpha = \arccos \frac{p1 * p2}{|p1| * |p2|}$$

The angle between these vectors is

$\alpha = 58.826°$

Then we have to find cross product between the vectors. It can be found

$$v = p1 \text{ x } p2 = (\begin{vmatrix} p1(2) & p1(3) \\ p2(2) & p2(3) \end{vmatrix}, -\begin{vmatrix} p1(1) & p1(3) \\ p2(1) & p2(3) \end{vmatrix}, \begin{vmatrix} p1(1) & p1(2) \\ p2(1) & p2(2) \end{vmatrix})$$

p1 x p2 is perpendicular to both vectors.

$$v = \begin{pmatrix} -0.32105 \\ -0.32105 \\ -0.72519 \end{pmatrix}$$

It has to be taken norm of v, which means

$$v = \frac{p1 \text{ x } p2}{|p1| * |p2|}$$

The result of v is then

$$v = \begin{pmatrix} -0.37523 \\ -0.37523 \\ -0.84759 \end{pmatrix}$$

v shows, how much we have to rotate a vector in all three axes

It is now possible to find a quaternion. It can be computed:

$$q=\begin{pmatrix} \cos\frac{\alpha}{2} \\ v * \sin\frac{\alpha}{2} \end{pmatrix}$$

$$q = \begin{pmatrix} 0.8711 \\ -0.19144 \\ -0.19144 \\ -0.42151 \end{pmatrix}$$

Inversion of q is

$$q^{-1}=\begin{pmatrix} \cos\frac{\alpha}{2} \\ -v * \sin\frac{\alpha}{2} \end{pmatrix}$$

$$q^{-1} = \begin{pmatrix} 0.8711 \\ 0.19144 \\ 0.19144 \\ 0.42151 \end{pmatrix}$$

q and q^{-1} can be rewritten

$q = 0.8711 - 0.19144i – 0.19144j – 0.42151k$

$q^{-1} = 0.8711 + 0.19144i + 0.19144j + 0.42151k$

Now that we have found quaternion and inversion of quaternion, it is now possible to find the rotated vector, p', that can be found

$$p' = q*p1*q^{-1}$$

When using quaternions in computations, it is important to keep in mind that they are noncommutative, which means $q*q^{-1} \neq q^{-1}*q$.

In the next computation, i, j, k, are included, so we have to know, that

*	i	j	k
i	-1	k	-j
j	-k	-1	i
k	j	-i	-1

Table 3 Quaternion multiplication

p’ = (0.8711 - 0.19144i – 0.19144j – 0.42151k)(-0.17482i – 0.86934j + 0.46226k)(0.8711 + 0.19144i + 0.19144j + 0.42151k)

The same result can be found and in this example, using rotation matrix,R

$$R=\begin{pmatrix} a^2+\ b^2-c^2-d^2 & 2(-ad+bc) & 2(ac+bd) \\ 2(ad+bc) & a^2-\ b^2+c^2-d^2 & 2(-ab+cd) \\ 2(-ac+bd) & 2(ab+cd) & a^2-\ b^2-c^2+d^2 \end{pmatrix}$$

where a,b,c,d are components of v vector.

So we get

p’ = R = $\begin{pmatrix} -0.8833 \\ -0.16087 \\ 0.46214 \end{pmatrix}$

The desired vector is p2 = $\begin{pmatrix} -0.86934 \\ -0.17482 \\ 0.46226 \end{pmatrix}$

p2 and p’ are supposed to be equal. They are almost the same, but still differ little bit. The code in Matlab might have rounded results a little much. But these equations are suitable for rotating a vector to desired position.

4.2 Future work

As we have two vectors, measured from the sun sensors and calculated from the sun vector equations, it is possible to rotate a desired vector to desired position using quaternions. It is implemented in previous section. If we now have desired vector in desired position,we know, where is Earth with respect to the satellite and it is possible to determine satellite’s attitude. For the final satellite rotation, we must do the following: firstly, the satellite must be moved to the center of the Earth. Then it is possible to rotate satellite and then satellite has to be moved back to the origin position.

To get even more accurate results in satellite’s attitude, Earth Albedo model has to be implemented. It eliminates the sunlight reflected from Earth.

5 Summary

The purpose of this work was to determine satellite's attitude in orbit using sun sensors. This work provides sun position calculations, where the inputs are location (latitude and longitude) and timestamp. The output is the position of the Sun in horizontal and equatorial coordinate system. The coordinates were validated and compared using other sources, where the angles were computed.

Using the coordinates of the Sun , it was possible to calculate the sun vector, that gave the position of the Sun with respect to all three axes.

Having calculated the sun vector, the main goal – attitude of the satellite – was possible to calculate. In this work, quaternions were used to represent vector rotation. As we have two vectors, we have to rotate one vector toward the other to find out, where is Earth with respect to the satellite.

The calculations were implemented in Matlab. The code provides the calculations for finding the position of the sun horizontal and equatorial coordinates. Also, the sun vector calculation. And finally, the vector rotation using quaternions.

The main goal of this work is solved and formulas to determine satellite's attitude are accurate. There were slight differences in calculations, for example, finding the angles in equatorial coordinates. There was a slight difference in vector rotation. But they were still close to the real answers.

It is possible to develop this work. For example, adding the Earth Albedo model, which gives even more accurate answers.

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